MENIIT
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JEE Advanced : Paper-I (2019)

IMPORTANT INSTRUCTIONS

– This section contains FOUR (04) questions.
– Each question has FOUR options. ONLY ONE of these four options is the correct answer.
– For each question, choose the option corresponding to the correct answer.
– Answer to each question will be evaluated according to the following marking scheme :

Full Marks : +3 If ONLY the correct option is chosen.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered)

Negative Marks : –1 In all other cases

PART-A : PHYSICS

SECTION-1 : (Maximum Marks : 12)

– This section contains FOUR (04) questions.

– Each question has FOUR options. ONLY ONE of these four options is the correct answer.

– For each question, choose the option corresponding to the correct answer.

– Answer to each question will be evaluated according to the following marking scheme :

Full Marks : +3 If ONLY the correct option is chosen.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered)

Negative Marks : –1 In all other cases

1. A current carrying wire heats a metal rod. The wire provides a constant power (P) to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as :

$$T(t) = T_0\,(1 + \beta t^{1/4})$$

where β is a constant with appropriate dimension while T_0 is a constant with dimension of temperature. The heat capacity of the metal is :

(A) $\dfrac{4P(T(t)-T_0)^4}{\beta^4 T_0^5}$ (B) $\dfrac{4P(T(t)-T_0)^2}{\beta^4 T_0^3}$ (C) $\dfrac{4P(T(t)-T_0)}{\beta^4 T_0^2}$ (D*) $\dfrac{4P(T(t)-T_0)^3}{\beta^4 T_0^4}$

Ans. 4

Sol. $P = \dfrac{dQ}{dt}$ $T_{(t)} = T_0(1 + \beta t^{1/4})$

$$\frac{dQ}{dt} = \boxed{ms}\,\frac{dT}{dt} \Rightarrow s = \frac{P}{\left(\frac{dT}{dt}\right)} b$$

$$\frac{dT}{dt} = T_0\left[0 + \beta\frac{1}{4}\cdot t^{-3/4}\right] = \frac{\beta T_0}{4}\cdot t^{-3/4}$$

$$S = \frac{P}{(dT/dt)} = \frac{4P}{\beta T_0}\cdot t^{3/4}$$

$$S = \frac{4P}{\beta}\left[\frac{t^{3/4}}{T_0}\right]$$

$$\frac{T(t)}{T_0}(1+\beta t^{1/4})$$

$$\beta t^{1/4} = \frac{T(t)}{T_0} - 1 = \frac{T(t)-T_0}{T_0}$$

$$t^{3/4} = \left(\frac{T(t)-T_0}{\beta\cdot T_0}\right)^3$$

$$\Rightarrow S = \frac{4P}{T_0\beta}\left[\frac{T(t)-T_0}{\beta T_0}\right]^3 = \frac{4P}{\beta^4 T_0^4}[T(t)-T_0]^3$$

2. Consider a spherical gaseous cloud of mass density $\rho(r)$ in free space where r is the radial distance from its center. The gaseous cloud is made of particles of equal mass m moving in circular orbits about the common center with the same kinetic energy K. The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant in time, the particle number density $n(r) = \rho(r)/m$ is: [G is universal gravitational constant]

(A) $\frac{3K}{\pi r^2 m^2 G}$ (B*) $\frac{K}{2\pi r^2 m^2 G}$ (C) $\frac{K}{\pi r^2 m^2 G}$ (D) $\frac{K}{6\pi r^2 m^2 G}$

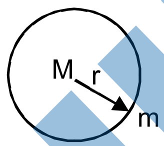
Ans. 2

Sol. Let total mass included in a sphere of radius r be M.

For a particleof mass m,

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\Rightarrow \frac{GMm}{r} = 2K \Rightarrow M = \frac{2Kr}{Gm}$$



$$\therefore dM = \frac{2Kdr}{Gm}$$

$$\Rightarrow (4\pi r^2 dr)\rho = \frac{2Kdr}{Gm}$$

$$\Rightarrow \rho = \frac{K}{2\pi r^2 Gm}$$

$$\therefore n = \frac{\rho}{m} = \frac{K}{2\pi r^2 m^2 G}$$

3. In a radioactive sample, ${}^{40}_{19}K$ nuclei either decay into stable ${}^{40}_{20}Ca$ nuclei with decay constant 4.5×10^{-10} per year or into stable ${}^{40}_{18}Ar$ nuclei with decay constant 0.5×10^{-10} per year. Given that in this sample all the stable ${}^{40}_{18}Ar$ and ${}^{40}_{18}Ar$ nuclei are produced by the ${}^{40}_{19}K$ nuclei only. In time $t \times 10^9$ years, if the ratio of the sum of stable ${}^{40}_{20}Ca$ and ${}^{40}_{18}Ar$ nuclei to the radioactive ${}^{40}_{19}K$ nuclei is 99, the value of t will be : [Given ln 10 = 2.3]

(A) 2.3 (B) 1.15 (C*) 9.2 (D) 4.6

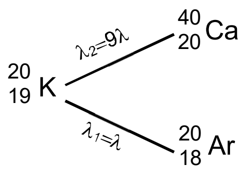
Ans. 3

Sol. Parallel radioactive decay

$\lambda = \lambda_1 + \lambda_2 = 5 \times 10^{-10}$ per year

$N = N_0 e^{-\lambda t}$

$N_0 - N = N_{stable}$



$N = N_{radioactive}$

$$\frac{N_0}{N} - 1 = 99$$

$$\frac{N_0}{N} = 100$$

$$\frac{N}{N_0} = e^{-\lambda t} = \frac{1}{100}$$

$\Rightarrow \lambda t = 2 \ln 10$

$= 4.6$

$t = 9.2 \times 10^9$ year

4. A thin spherical insulating shell of radius R carries a uniformly distributed charge such that the potential at its surface is V_0. A hole with a small area $\alpha 4\pi R^2$ (α<<1) is made on the shell without affecting the rest of the shell. Which one of the following statements is correct ?

(A) The potential at the center of the shell is reduced by $2\alpha V_0$

(B) The magnitude of electric field at a point, located on a line passing through the hole and shell's center on a distance 2R from the center of the spherical shell will be reduced by $\frac{\alpha V_0}{2R}$

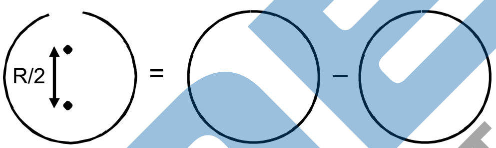
(C*) The ratio of the potential at the center of the shell to that of the point at $\frac{1}{2}$ R from center towards the hole will be $\frac{1-\alpha}{1-2\alpha}$

(D) The magnitude of electric field at the center of the shell is reduced by $\frac{\alpha V_0}{2R}$

Sol. Let charge on the sphere initially be

$$\therefore \frac{kQ}{R} - V_0$$

and charge removed = αQ

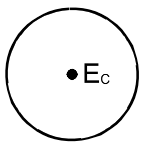


and $V_P = \frac{kQ}{R} - \frac{2K\alpha Q}{R} = \frac{kQ}{R}(1-2\alpha)$

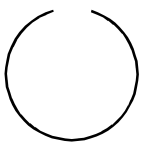
$$V_C = \frac{kQ(1-\alpha)}{R}$$

$$\therefore \frac{V_C}{V_P} = \frac{1-\alpha}{1-2\alpha}$$

(B) $(E_C)_{initial}$ = zero

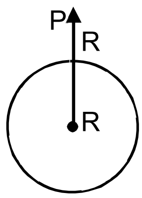


$$(E_C)_{final} = \frac{k\alpha Q}{R^2}$$

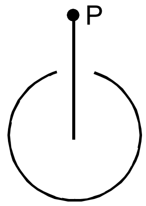


$\Rightarrow$ Electric field increases

(C) $(E_P)_{initial} = \frac{kQ}{4R^2}$

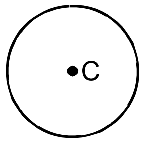


$(E_P)_{final} = \frac{kQ}{4R^2} - \frac{k\alpha Q}{R^2}$



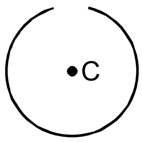
$\Delta E_P = \frac{kQ}{4R^2} - \frac{kQ}{4R^2} + \frac{k\alpha Q}{R^2} = \frac{V_0 \alpha}{R}$

(D) $(VC)_{initial} = \frac{kQ}{R}$



$(V_C)_{final} = \frac{kQ(1-\alpha)}{R}$

$\Delta V_C = \frac{kQ}{R}(\alpha) = \alpha V_0$



SECTION-2 : (Maximum Marks: 32)

- This section contains EIGHT (08) questions.
- Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s)
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is (are) chosen.

Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.

Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct.

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : –1 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;

choosing ONLY (A) and (B) will get +2 marks;

choosing ONLY (A) and (D) will get +2 marks;

choosing ONLY (A) will get +1 marks;

choosing ONLY (B) will get +1 marks;

choosing ONLY (D) will get +1 marks;

choosing no option (i.e. the question is unanswered) will get 0 marks, and

choosing any other combination of options will get –1 mark.

5. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following statement (s) is/are correct ?

(A*) The dimension of energy is L^{-2}

(B*) The dimension of linear momentum is L^{-1}

(C*) The dimension of force is L^{-3}

(D) The dimension of power is L^{-5}

Sol. Mass = $M^0L^0T^0$

MVr = $M^0L^0T^0$

$$M^0 \frac{L'}{T'} \cdot L' = M^0L^0T^0$$

$L^2 = T^1$ .... (A)

Force $= M^1L^{-1}T^{-2}$ (in SI)

$= M^0L^1L^{-4}$ (In new system from equation (A))

$= L^{-3}$

Energy $= M^1L^2T^{-2}$ (in SI)

$= M^0L^2L^{-4}$ (In new system from equaiton (A))

$= L^{-2}$

$$\text{Power} = \frac{\text{Energy}}{\text{Time}}$$

$= M^1L^2T^{-3}$ (in SI)

$= M^0L^2L^{-6}$ (In new system from equation (A))

$= L^{-4}$

Linear momentum $= M^1L^1T^{-1}$ (in SI)

$= M^0L^1L^{-2}$ (In new system from equation (A))

$= L^{-1}$

6. Two identical moving coil galvanometer have 10 Ω resistance and full scale deflection at 2 μA current. One of them is converted into a voltmeter of 100 mV full scale reading and the other into an Ammeter of 1 mA full scale current using appropriate resistors. These are then used to measure the voltage and current in the Ohm's law experiment with R = 1000 Ω resistor by using an ideal cell. Which of the following statement(s) is/are correct ?

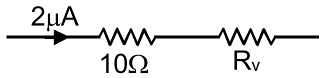
(A) The resistance of the Voltmeter will be 100 kΩ.

(B*) The measured value of R will be 978 Ω < R < 982 Ω.

(C) If the ideal cell is replaced by a cell having internal resistance of 5 Ω then the measured value of R will be more than 1000 Ω.

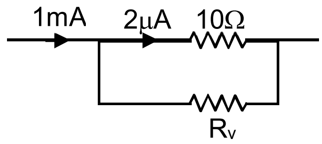
(D*) The resistance of the Ammeter will be 0.02 Ω (round off to 2nd decimal place)

Sol.



$0.1 = 2 \times 10^{-6}\,(10 + R)$

$\therefore R_v = 49990\ \Omega$



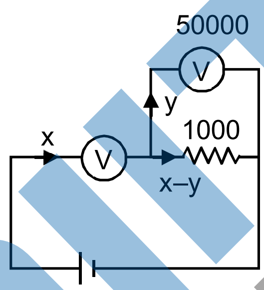
$2 \times 10^{-6} \times 10 = 10^{-3} R_A$

$\therefore R_A = 0.02\ \Omega$

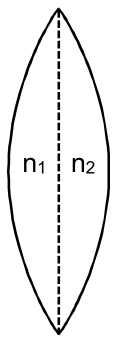
$y\,5000 = (x - y)\,1000$

$\therefore 51y = x$

Reading $= \dfrac{y\,50000}{x} \simeq 980$



7. A thin convex lens is made of two materials with refractive indices n_1 and n_2, as shown in figure. The radius of curvature of the left and right spherical surfaces are equal. f is the focal length of the lens when $n_1 = n_2 = n$. The focal length is $f + \Delta f$ when $n_1 = n$ and $n_2 = n + \Delta n$. Assuming $\Delta n << (n-1)$ and $1 < n < 2$, the correct statement(s) is/are :



(A) $\left|\dfrac{\Delta f}{f}\right| < \left|\dfrac{\Delta n}{n}\right|$

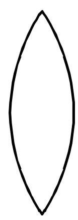
(B*) For $n = 1.5$, $\Delta n = 10^{-3}$ and $f = 20$ cm, the value of $|\Delta f|$ will be 0.02 cm (round off to 2nd decimal place)

(C*) If $\dfrac{\Delta n}{n} < 0$ then $\dfrac{\Delta f}{f} > 0$

(D*) The relation between $\dfrac{\Delta f}{f}$ and $\dfrac{\Delta n}{n}$ remains unchanged if both the convex surfaces are replaced by concave surfaces of the same radius of curvature

Sol. When $n_1 = n_2 = n$

$$\frac{1}{f} = (n-1) \times \frac{2}{R}$$



So $\quad f = \dfrac{R}{2(n-1)} \quad$ ... (A)

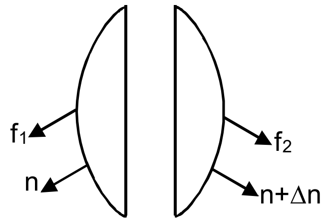
2nd case :

$$\frac{1}{f_1} = \frac{n-1}{R}$$

$$\frac{1}{f_2} = \frac{(n+\Delta n)-1}{R}$$

$$\frac{1}{f_{eq}} = \frac{1}{f+\Delta f} = \left(\frac{n-1}{R}\right) + \frac{(n+\Delta n)-1}{R} = \frac{2(n-1)+\Delta n}{R}$$

$$\Delta f = \left(\frac{R}{2(n-1)+\Delta n}\right) - \left(\frac{R}{2(n-1)}\right)$$



$$= \frac{R}{2}\left[\frac{(n-1)-(n-1+\Delta n)}{(n-1+\Delta n)(n-1)}\right] = \frac{-\Delta n}{(n-1)^2} \times \frac{R}{2}$$

$$\frac{\Delta f}{f} = -\frac{\Delta n}{2(n-1)} \qquad \text{... (B)}$$

(A) Relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ is independent of R so (A) is correct.

(B) 2n – 2 < n because n < 2

$$\Rightarrow \frac{\Delta f}{f} = \frac{1}{2}\left|\frac{\Delta n}{n-1}\right| > \frac{\Delta n}{n}$$

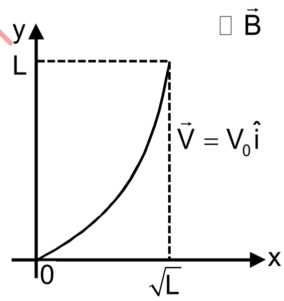
So $\frac{\Delta f}{f} > \left|\frac{\Delta n}{n}\right|$ So (B) is wrong

(C) $|\Delta f| = \frac{f\Delta n}{(n-1)} = \frac{(20\times10^{-3})}{1.5-1} = 40\times10^{-3} = 0.04$

So (C) is wrong

(D) If $\frac{\Delta n}{n} < 0$ then $\frac{\Delta f}{f} > 0$ from equation (B)

8. A conducting wire of parabolic shape, initially $y = x^2$, is moving with velocity $\vec{V} = V_0\hat{i}$ in a non-uniform magnetic field $\vec{B} = B_0\left(1+\left(\frac{y}{L}\right)^{\beta}\right)\hat{k}$, as shown in figure. If V_0, B_0, L and β are positive constants and Δφ is the potential difference developed between the ends of the wire, then the correct statement(s) is/are:



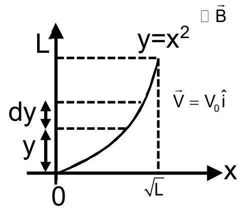
(A*) |Δφ| remains the same if the parabolic wire is replaced by a straight wire, y = x initially, of length $\sqrt{2}L$

(B*) $|\Delta\varphi| = \frac{4}{3}B_0V_0L$ for β = 2

(C*) |Δφ| is proportional to the length of the wire projected on the y-axis.

(D) $|\Delta\varphi| = \frac{1}{2}B_0V_0L$ for β = 0

Sol. $y = x^2$



$$B = B_0\left[1+\left(\frac{y}{L}\right)^{\beta}\right]\hat{k}$$

$$\int d\varphi \int_0^L V_0B_0\left(1+\frac{y^{\beta}}{L^{\beta}}\right)\cdot dy$$

$$\Delta\varphi = V_0B_0\left[L+\frac{L^{\beta+1}}{(\beta+1)L^{\beta}}\right]$$

$$\Delta\varphi = V_0B_0\left[L+\frac{L}{\beta+1}\right]$$

$$\because |\Delta\varphi| = B_0V_0\left(1+\frac{1}{\beta+1}\right)\cdot L$$

$|\Delta\phi| \propto L$

$\therefore$ option '2' is also correct

If $\beta = 0$

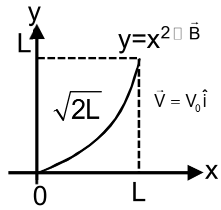
$\Delta\phi = V_0B_0\,[L + L]$

$\Delta\phi = 2V_0B_0L \Rightarrow$ option (C) is incorrect

If $\beta = 2$

$$\Delta\varphi = B_0V_0\left[L+\frac{L}{3}\right]$$

$$\Delta\varphi = \frac{4}{3}V_0B_0L$$ option (D) is correct

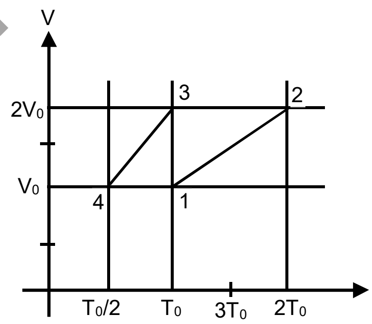


$\Delta\phi$ will be same if the wire is replaced by the straight wire of length $\sqrt{2}L$ and $y = x$

range of y remains same

$\therefore$ option 1 is correct.

9. One mole of a monoatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V-T) diagram. The correct statement(s) is/are : [R is the gas constant]



(A) The ratio of heat transfer during processes $1 \to 2$ and $3 \to 4$ is $\left|\frac{Q_{1\to2}}{Q_{3\to4}}\right| = \frac{1}{2}$

(B) The above thermodynamic cycle exhibits only isochoric and adiabatic processes

(C*) The ratio of heat transfer during processes 1→ 2 and 2 → 3 is $\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \frac{5}{3}$

(D*) Work done in this thermodynamic cycle (1 → 2 → 3 → 4 → 1) is $|W| = \frac{1}{2}RT_0$

Sol. From graph

Process 1 → 2 is isobaric with $P = \frac{RT_0}{V_0}$

Process 2 → 3 is isobaric with $V = 2V_0$

Process 3 → 4 is isobaric with $P = \frac{RT_0}{2V_0}$

Process 4 → 1 is isobaric with $V = V_0$

Work in cycle = $\frac{RT_0}{V_0}\cdot V_0 - \frac{RT_0}{2V_0}\cdot V_0 = \frac{RT_0}{2}$

$Q_{1-2} = nC_P\Delta T = n\cdot\frac{5R}{2}\cdot T_0$

$Q_{2-3} = nC_V\Delta T = n\cdot\frac{n}{2}\cdot T_0$

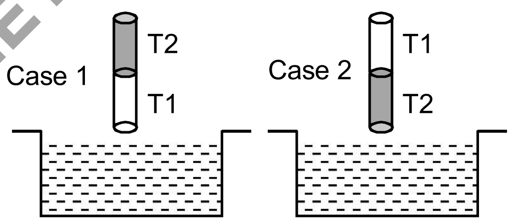
$\therefore \left|\frac{Q_{1-2}}{Q_{2-3}}\right| = \frac{5}{3}$

$Q_{3-4} = nC_P\Delta T = n\cdot\frac{5R}{2}\cdot\frac{T_0}{2}$

$\therefore \left|\frac{Q_{1-2}}{Q_{3-4}}\right| = 2$

10. A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries T1 and T2 of different materials having water contact angles of 0° and 60°, respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is(are) correct ?

(Surface tension of water = 0.075 N/m, density of water = 1000 kg/m^3, take g = 10 m/s^2)



(A) For case I, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. (Neglect the weight of the water in the meniscus)

(B*) The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be different for both cases.

(C*) For case I, if the joint is kept at 8 cm above the water surface, the height of water column in the tube will be 7.5 cm. (Neglect the weight of the water in the meniscus)

(D*) For case II, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be 3.75 cm. (Neglect the weight of the water in the meniscus)

Sol. $h = \frac{2T\cos\theta}{\rho g R}$; $h_1 = \frac{2 \times 0.075 \times \cos 0°}{1000 \times 10 \times 0.2 \times 10^{-3}}$

$\Rightarrow h_1$ = 75 mm (in T_1) [If we assume entire tube of T_1]

$\Rightarrow h_2 = \frac{2 \times 0.075 \times \cos 60°}{1000 \times 10 \times 0.2 \times 10^{-3}}$ = 37.5 mm (in T_2) [If we assume entire tube of T_2]

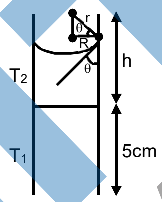
Option (A) : Since contact angles are different so correction in the height of water column raised in the tube will be different in both the cases.

Option (B) : If joint is 5 cm is above water surface, then lets say water crosses the joint by height h, then:

$\Rightarrow P_0 - \frac{2T}{r} + \rho g h + \rho g \times 5 \times 10^{-2}$

$= P_0$

$\Rightarrow \cos\theta = \frac{R}{r} \cdot r = \frac{R}{\cos\theta}$



$\Rightarrow \rho g(h + 5 \times 10^{-2}) = \frac{2T\cos\theta}{R}$

$\Rightarrow h = \frac{2 \times 0.075 \times \cos 60°}{0.2 \times 10^{-3} \times 1000 \times 10} = 5 \times 10^{-2}$

$\Rightarrow$ h = –ve, not possible, so liquid will not cross the interface, but angle of contact at the interface will change, to balance the pressure,

So option (B) is wrong.

Option (C) : If interface is 8 cm above water then water will not even reach the interface, and water will rise till 7.5 cm only in T1, so option (C) is right.

Option (D) : If interface is 5 cm above the water in vessel, then water in capillary will not even reach the interface. Water will reach only till 3.75 cm, so option (D) is right.

11. A charged shell of radius R carries a total charge Given Φ as the flux of electric field through a closed cylindrical surface of height h, radius r and with its center same as that of the shell. Here, center of the cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Which of the following option(s) is/are correct ?

[ϵ_0 is the permittivity of free space]

(A*) If $h < \frac{8R}{5}$ and $r = \frac{3R}{5}$ then $\Phi = 0$

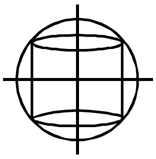
(B*) If h > 2R and $r = \frac{3R}{5}$ then $\Phi = \frac{Q}{5\epsilon_0}$

(C*) If h > 2R and r > R then $\Phi = \frac{Q}{\epsilon_0}$

(D) If h > 2R and $r = \frac{4R}{5}$ then $\Phi = \frac{Q}{5\epsilon_0}$

Sol. For option (A), cylinder encloses the shell, thus option is correct

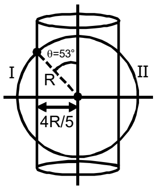
For option (B),



cylinder perfectly enclosed by shell,

thus $\phi = 0$, so option is correct.

For option (C)

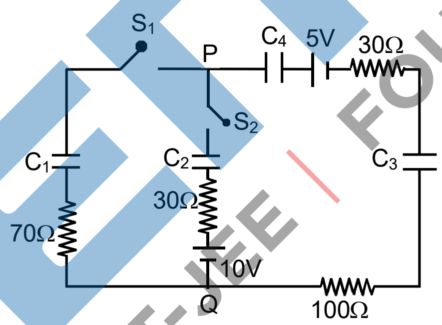


$$\varphi = \frac{2\times Q}{2\in_0}(1-\cos 53°) = \frac{2Q}{5\in_0}$$

For option (D) :

Flux enclosed by cylinder = $\varphi = \frac{2Q}{2\in_0}(1-\cos 37°) = \frac{Q}{5\in_0}$

12. In the circuit shown, initially there is no charge on capacitors and keys S_1 and S_2 are open. The values of the capacitors are C_1 = 10 μF, C_2 = 30 μF and $C_3 = C_4$ = 80 μF.



Which of the statement(s) is/are correct ?

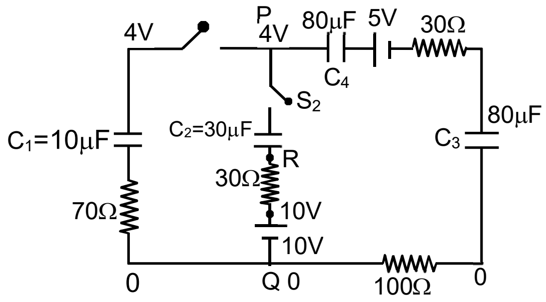
(A*) At time t = 0, the key S1 is closed, the instantaneous current in the closed circuit will be 25 mA.

(B) If key S1 is kept closed for long time such that capacitors are fully charged, the voltage difference between points P and Q will be 10 V.

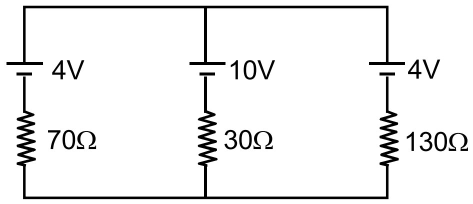
(C*) If key S1 is kept closed for long time such that capacitors are fully charged, the voltage across the capacitors C1 will be 4V.

(D) The keys S1 is kept closed for long time such that capacitors are fully charged. Now key S2 is closed, at this time, the instantaneous current across 30 Ω resistor (between points P and Q) will be 0.2 A (round off to 1^{st} decimal place).

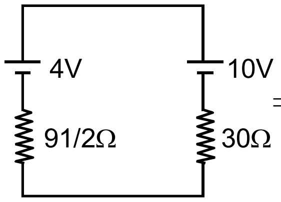
Sol.



(A) at t = 0, capacitor C_1 acts as a battery of 4V, C_4 & C_3 of $\frac{1}{2}$V each, C_2 is shorted circuit is



⇒



$\Rightarrow i = \dfrac{6}{30+\frac{91}{2}} = 0.079A$

(B) and (D)

At steady state,

When capacitor is fully charged it behave as open circuit and current through it zero.

Hence, Charge on each capacitor is same.

$Q = C_{eq}V$

$= (8\ \mu F) \times 5$

$Q = 40\ \mu C$

Now, $V_P = \dfrac{40}{10} = V_0$

$V_P - V_Q = 4V$

(C) At t = 0, S_1 is closed, capacitor act as short circuit.

$= \dfrac{V}{R_{eq}} = \dfrac{5}{200} = 25mA$

SECTION-3 : (Maximum Marks: 18)

- This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/round-off the value to Two decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If ONLY the correct numerical value is entered.

  Zero Marks : 0 In all other cases.

13. A liquid at 30°C is poured very slowly into a Calorimeter that is at temperature of 110°C. The boiling temperature of the liquid is 80°C. It is found that the first 5 gm of the liquid completely evaporates. After pouring another 80 gm of the liquid the equilibrium temperature is found to be 50°C. The ratio of the Latent heat of the liquid to its specific heat will be ______ °C.

[Neglect the heat exchange with surrounding]

Ans. 270.00

Sol. Let m = mass of calorimeter

x = specific heat of calorimeter

s = specific heat of liquid

L = latent heat of liquid

First 5gof liquid at 30° is poured to calorimeter at 110°C

$\therefore$ m × x × (110 – 80) = 5 × s × (80 × 30) + 5L

$\Rightarrow$ mx × 30 = 250 s + 5L ... (A)

Now, 80 g of liquid at 30° is poured into colarimeter at 80°C, then equilibrium temperature reaches to 50°C.

$\therefore$ m × x × (80 – 30) = 80 × s × (50 – 30)

$\Rightarrow$ mx × 30 = 1600 s .... (ii)

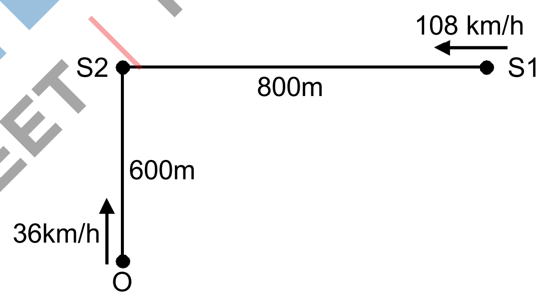
from (i) & (ii)

250 s + 5L = 1600 s $\Rightarrow$ 5L = 1350 s

$$\Rightarrow \frac{L}{s} = 270$$

14. A train S1, moving with a uniform velocity of 108 km/h, approaches another train S2 standing on a platform. An observer O moves with a uniform velocity of 36 km/h towards S2, as shown in figure. Both the trains are blowing whistles of same frequency 120 Hz. When O is 600 m away from S2 and distance between S1 and S2 is 800 m, the number of beats heard by O is ____.

[Speed of the sound = 330 m/s]



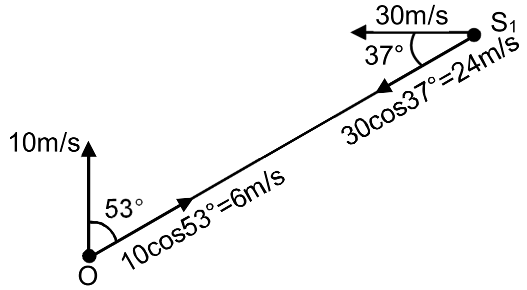
Ans. 8 (7.00 to 10.00)

Sol. Frequency observed by O from S_2

$$f_2 = \frac{330+10}{330} \times 120 = 123.63\text{Hz}$$

frequency observed by O from S_1

$$f_1 = \frac{330+6}{330-24} \times 120 = \frac{336}{306} \times 120 \approx 131.76\text{Hz}$$

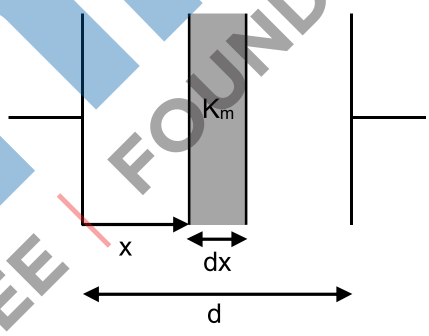


beat frequency = 131376 – 123.63 = 8.128 ≈ 8.12 to 8.13 Hz

15. A parallel plate capacitor of capacitance C has spacing d between two plates having area A. The region between the plates is filled with N dielectric layers, parallel to its plates, each with thickness $\delta = \frac{d}{N}$. The dielectric constant of the m^{th} layer is $K_m = K\left(1+\frac{m}{N}\right)$. For a very large N (> 10^3), the capacitance C is $\alpha\left(\frac{K \in_0 A}{d\ln 2}\right)$. The value of a will be ______. [$\in_0$ is the permittivity of free space]

Ans. 1.00 (0.99 to 1.01)

Sol. $\delta = dx = \frac{d}{N}$ & $\frac{m}{N} = \frac{x}{d}$



$$K_m = K\left(1+\frac{m}{N}\right)$$

$$\Rightarrow K_m = K\left(1+\frac{x}{d}\right)$$

$$C' = \frac{K_m A \in_0}{dx}$$

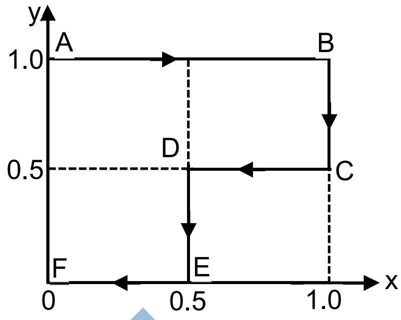
$$\frac{1}{C_{eq}} = \int_0^d \frac{dx}{K_m A \in_0} = \frac{1}{KA\in_0}\int_0^d \frac{dx}{\left(1+\frac{x}{d}\right)}$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{d}{KA\in_0}\left[\ln\left(1+\frac{x}{d}\right)\right]_0^d$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{d}{KA\in_0}[\ln 2 - \ln(1)]$$

$$\Rightarrow C_{eq} = \frac{KA\in_0}{d\ln 2} \Rightarrow \alpha = 1$$

16. A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a force $\vec{F} = (\alpha y\hat{i} + 2\alpha x\hat{j})N$ where x and y are in meter and α = –1 N/m^{-1}. The work done on the particle by this force $\vec{F}$ will be ____ Joule.



Ans. 0.75

Sol. $F = (\alpha y\hat{i} + 2\alpha x\hat{j})$

$W_{AB} = (-1\hat{i})\cdot(1\hat{i}) = -1J$

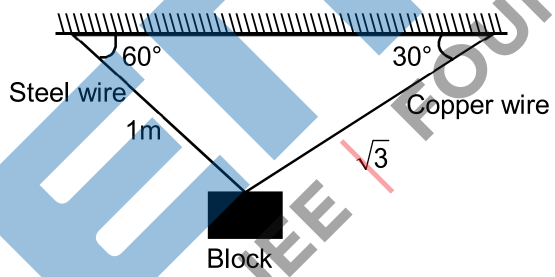
$\begin{bmatrix} \vec{F} = -1\hat{i} + 2\alpha x\,\hat{j} \\ \vec{S} = 1\hat{i} \end{bmatrix}$

Similarly,

W_{BC} = 1J

17. A block of weight 100 N is suspended by copper and steel wires of same cross sectional area 0.5 cm^2 and, length $\sqrt{3}$m and 1 m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with ceiling are 30° and 60°, respectively. If elongation in copper wire is ($\Delta\ell_C$) and elongation in steel wire is ($\Delta\ell_S$), then the ratio $\frac{\Delta\ell_C}{\Delta\ell_S}$ ____.

[Young's modulus for copper and steel are 1 × 10^{11} N/m^2 and 2 × 10^{11} N/m^2 respectively]



Ans. 2.00

Sol. Let T_S = tension in steel wire

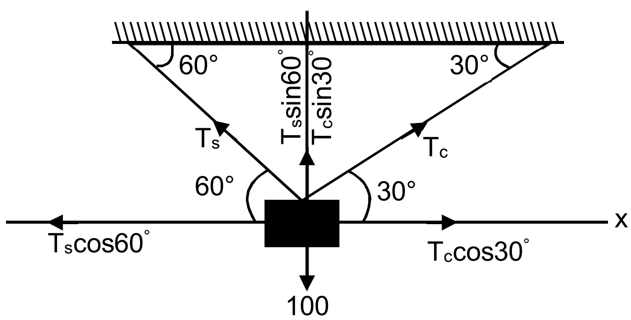
T_C = Tension in copper wire in x direction

T_C cos 30° = $T_S$cos 60°

$T_C \times \frac{\sqrt{3}}{2} = T_S \times \frac{1}{2}$

$\sqrt{3}T_C = T_S$ ... (A)

in y direction



T_C sin 30° + T_S sin 60° = 100

$\frac{T_C}{2} + \frac{T_S\sqrt{3}}{2} = 100$ ... (ii)

Solving equation (i) & (ii)

T_C = 50 N

$T_S = 50\sqrt{3}N$

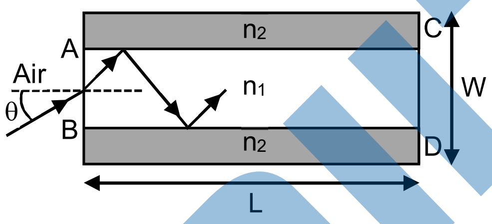
We know

$$\Delta L = \frac{FL}{AY} = \frac{\Delta L_C}{\Delta L_S} = \frac{T_C L_C}{A_C Y_C} \times \frac{A_S Y_S}{T_S L_S}$$

On solving above equation

$$\frac{\Delta L_C}{\Delta L_S} = 2$$

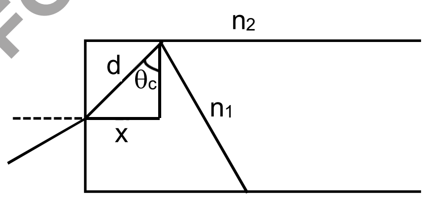
18 A planar structure of length L and width W is made of two different optical media of refractive indices n_1 = 1.5 and n_2 = 1.44 as shown in figure. If L >> W, a ray entering from end AB will emerge from end CD only if the total internal reflection condition is met inside the structure. For L = 9.6 m, if the incident angle θ is varied, the maximum time taken by a ray to exit the plane CD is $t \times 10^{-9}$ s, where t is ____. [Speed of light $c = 3 \times 10^8$ m/s]



Ans. 50 (49.00 to 51.00)

Sol. For maximum time the ray of lgith must undergo TIR at a all surface at minimum angle i.e. θC

For TIR $n_1 \sin\theta_C = n_2$



$$\sin\theta_C = \frac{1.44}{1.5}$$

In above Δ $\sin\theta_C = \frac{x}{d}$

$$d = \frac{x}{\sin\theta_C}$$

Similarly $D = \dfrac{L}{\sin\theta_C}$

where L = length of tube, D = length of path of light

Time taken by light

$$t = \frac{D}{C} = \frac{L/\sin\theta_C}{2\times10^8}$$

$t = 50 \times 10^{-9}$ s

PART-B: CHEMISTRY

SECTION-1 : (Maximum Marks : 12)

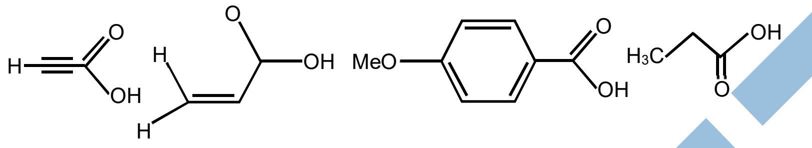
- This section contains FOUR (04) questions.
- Each question has FOUR options. ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :

  Full Marks : +3 If ONLY the correct option is chosen.

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered)

  Negative Marks : –1 In all other cases

19. The correct order of acid strength of the following carboxylic acids is -



(A) III > II > I > IV (B*) I > II > III > IV (C) II > I > IV > III (D) I > III > II > IV

20. The green colour produced in the borax bead test of a chromium(III) salt is due to -

(A*) $Cr(BO_2)_3$ (B) $Cr_2(B_4O_7)_3$ (C) CrB (D) Cr_2O_3

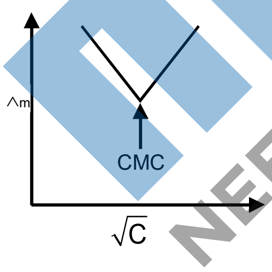
Sol. $Cr(BO_2)_3$

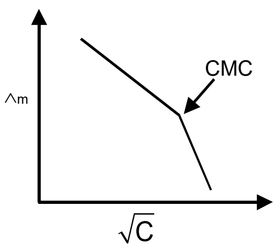
$$Na_2B_4O_7 \cdot 10H_2O \xrightarrow{\Delta} Na_2B_4O_7 \xrightarrow{\Delta} NaBO_2 + B_2O_3$$

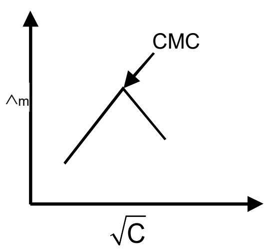
$$Cr_2O_3 + B_2O_3 \longrightarrow \underset{\text{green colour bead}}{Cr(BO_2)_3}$$

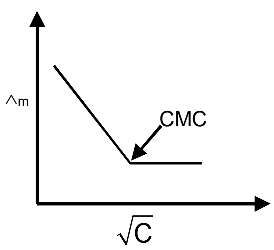
21. Molar conductivity ($\wedge_m$) of aqueous solution of sodium stearate, which behaves as a strong electrolyte, is recorded at varying concentration(c) of sodium stearate. Which one of the following plots provides the correct representation of micelle formation in the solution ?

(Critical micelle concentration (CMC) is marked with an arrow in the figures.)

(A)


(B*)


(C)


(D)


22. Calamine, malachite, magnetite and cryolite, respectively are

(A) $ZnCO_3$, $CuCO_3$, Fe_2O_3, Na_3AlF_6

(B*) $ZnCO_3$, $CuCO_3.Cu(OH)_2$, Fe_3O_4, Na_3AlF_6

(C) $ZnSO_4$, $Cu(OH)_2$, Fe_3O_4, Na_3AlF_6

(D) $ZnSO_4$, $CuCO_3$, Fe_2O_3, AlF_3]

Sol. Calamine – $ZnCO_3$

Malachite – $CuCO_3.Cu(OH)_2$

Magnetite – Fe_3O_4

Cryolite – Na_3AlF_6

SECTION-2 : (Maximum Marks: 32)

- This section contains EIGHT (08) questions.
- Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s)
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If only (all) the correct option(s) is (are) chosen.

  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.

  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct.

  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

  Negative Marks : –1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get +4 marks;

  choosing ONLY (A) and (B) will get +2 marks;

  choosing ONLY (A) and (D) will get +2 marks;

  choosing ONLY (B) and (D) will get +2 marks;

  choosing ONLY (A) will get +1 marks;

  choosing ONLY (B) will get +1 marks;

  choosing ONLY (D) will get +1 marks;

  choosing no option (i.e. the question is unanswered) will get 0 marks, and

  choosing any other combination of options will get –1 mark.

23. Which of the following statement(s) is (are) correct regarding the root mean square speed (U_{rms}) and average translational kinetic energy (ε_{av}) of a molecule in a gas at equilibrium ?

(A*) U_{rms} is inversely proportional to the square root of its molecular mass

(B) ε_{av} is doubled when its temperature is increased four times

(C*) ε_{av} at a given temperature does not depend on its molecular mass

(D*) U_{rms} is doubled when its temperature is increased four times

Sol. $E_{av} = \frac{3}{2}RT$ $\qquad U_{rms} = \sqrt{\frac{3RT}{M}}$

E_{av} does not depend on its molecular mass but depends upon absolute temperature.

24. Choose the reaction(s) from the following options, for which the standard enthalpy of reaction is equal to the standard enthalpy of formation.

(A*) $\frac{3}{2}\,O_2(g) \rightarrow O_3(g)$

(B) $2H_2(g) + O_2(g) \rightarrow 2H_2O(l)$

(C*) $\frac{1}{8}S_8 + O_2\,(g) \rightarrow SO_2(g)$

(D) $2C(g) + 3H_2(g) \rightarrow C_2H_6(g)$

Sol. Standard enthalpy of formation of a compound is the standard enthalpy when one mole of a compound is formed from the elements in their stable state of aggregation.

25. Each of the following options contains a set of four molecules. Identify the option(s) where all four molecules possess permanent dipole moment at room temperature.

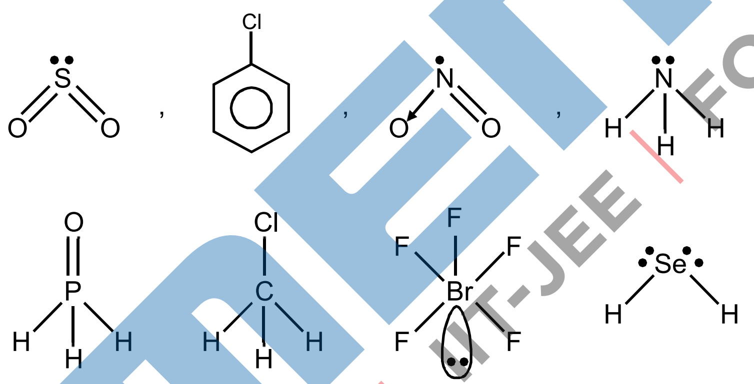
(A) BF_3, O_3, SF_6, XeF_6

(B) $BeCl_2$, CO_2, BCl_3, $CHCl_3$

(C*) NO_2, NH_3, $POCl_3$, CH_3Cl

(D*) SO_2, C_6H_5Cl, H_2Se, BrF_5

Sol.



26. A tin chloride Q undergoes the following reactions (not balanced)

$Q + Cl^{-} \rightarrow X$

$Q + Me_3N \rightarrow Y$

$Q + CuCl_2 \rightarrow Z + CuCl$

X is a monoanion having pyramidal geometry. Both Y and Z are neutral compounds. Choose the correct option(s).

(A) The oxidation state of the central atom in Z is +2

(B*) The central atoms in X is sp^3 hybridized

(C) The central atom in Z has one lone pair of electrons

(D*) There is a coordinate bond in Y

Sol. $\underset{(Q)}{SnCl_2} + Cl^{-} \longrightarrow \underset{(X)}{SnCl_3^{-}}$

$$\underset{(Q)}{SnCl_2} + Me_3N \longrightarrow \underset{(Y)}{SnCl_2[N(CH_3)_2]}$$

$$\underset{(Q)}{SnCl_2} + 2CuCl_2 \longrightarrow \underset{(Z)}{SnCl_4} + CuCl$$

27. In the decay sequence :

$$^{238}_{92}U \xrightarrow{-x_1} {}^{234}_{90}Th \xrightarrow{-x_2} {}^{234}_{91}Pa \xrightarrow{-x_3} {}^{234}Z \xrightarrow{-x_4} {}^{234}_{90}Th$$

x_1, x_2, x_3 and x_4 are particles/ radiation emitted by the respective isotopes. The correct option(s) is/are-

(A) x_3 is γ-ray

(B*) Z is an isotope of uranium

(C*) x_1 will deflect towards negatively charged plate

(D*) x_2 is β^-

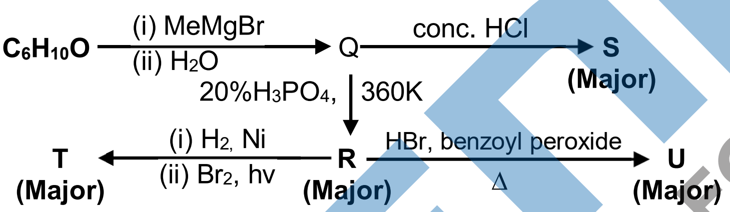
Sol. $X_1 = \alpha$

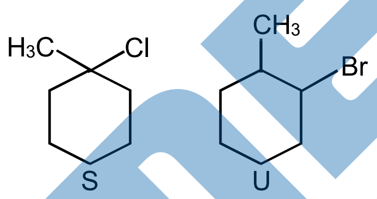
$X_2 = \beta$

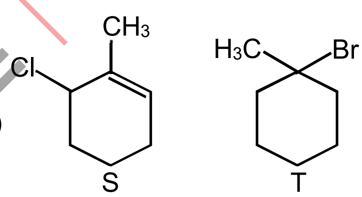
$X_3 = \beta$

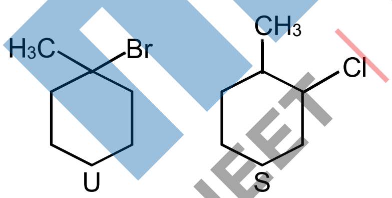
$X_4 = \alpha$

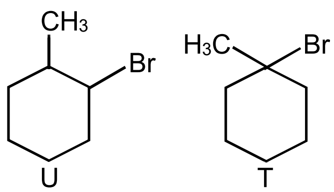
28. Choose the correct option(s) for the following set of reactions



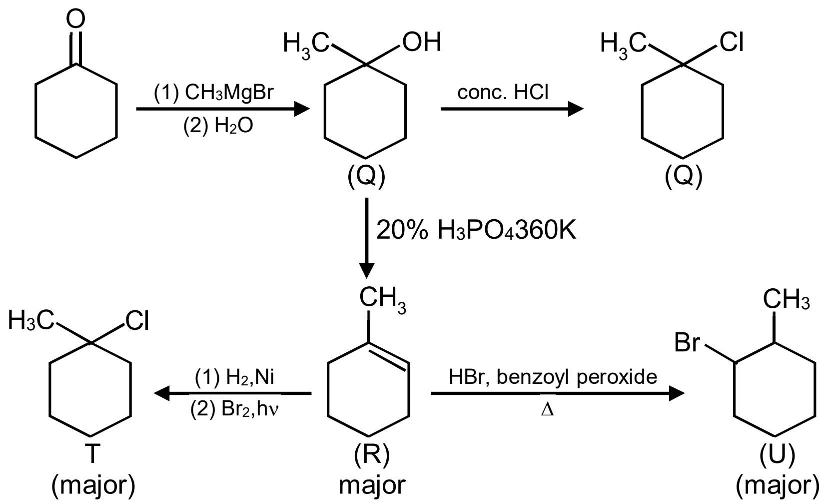
(A*)


(B)


(C)


(D*)


Sol.



29. Which of the following statement(s) is(are) true ?

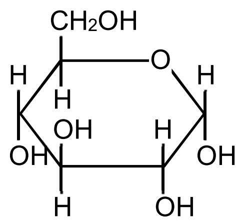
(A*) The two six-membered cyclic hemiacetal forms of D-(+)-glucose ard called anomers

(B*) Hydrolysis of sucrose gives dextrorotatory glucose and laevorotatory fructose

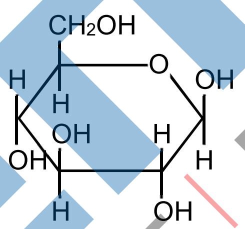
(C) Oxidation of glucose with bromine water gives glutamic acid

(D*) Monosaccharides cannot be hydrolysed to give polyhydroxy aldehydes and ketones

Sol. $\text{sucrose} \xrightarrow[\text{Hydrolysis}]{H^+, H_2O} \underset{\text{(Dextrorotatory)}}{D(+)\text{Glucose}} + \underset{\text{(laevoratatory)}}{D(-)\text{Fructose}}$



α - D - glucopyranose



β - D - glucopyranose

α-D-glucopyranose and β-D-glucopyranose are anomers of each other.

30. Fusion of MnO_2 with KOH in presence of O_2 produces a salt W. Alkaline solution of W upon eletrolytic oxidation yields another salt X. The manganese containing ions present in W and X, respectively, are Y and Z. Correct statement(s) is (are)

(A*) In both Y and Z, π-bonding occurs between p-orbitals of oxygen and d-orbitals of manganese.

(B*) Both Y and Z are coloured and have tetrahedral shape

(C) Y is diamagnetic in nature while Z is paramagnetic

(D*) In aqueous acidic solution, Y undergoes disproportionation reaction to give Z and MnO_2.

Sol. $MnO_2 \xrightarrow{KOH/O_2} \underset{w}{MnO_4^{2-}} \xrightarrow[\text{Oxidation}]{\text{Electrolytic}} MnO_4^-$

$MnO_4^{2-} \xrightarrow{\text{acidic medium}} MnO_2 + MnO_4^-$

$y = Mn^{+6}$ and $z = Mn^{+7}$

SECTION-3 : (Maximum Marks: 18)

- This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/round-off the value to Two decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If ONLY the correct numerical value is entered.

  Zero Marks : 0 In all other cases.

31. For the following reaction, the equilibrium constant K_c at 298 K is 1.6×10^{17}.

$$Fe^{2+}(aq) + S^{2-}(aq) \rightleftharpoons FeS(s)$$

When equal volumes of 0.06 M $Fe^{2+}(aq)$ and 0.2 M $S^{2-}(aq)$ solutions are mixed, the equilibrium concentration of $Fe^{2+}(aq)$ is found to be $Y \times 10^{-17}$ M. The value of Y is ________

Ans. 8.93

Sol.

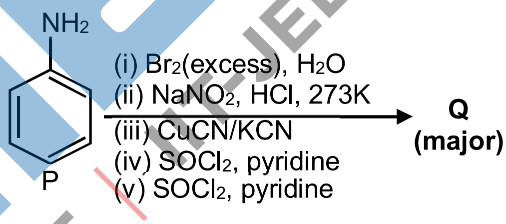
	Fe^{2+} aq) +	$S^{2-}(aq) \rightleftharpoons FeS(s)$	$K_c = 1.6 \times 10^{17}$
Initial	0.06 M	0.02 M	
After mixing	0.03 M	0.01 M	
	?	0.07 M	

$$1.6\times10^{17} = \frac{1}{[Fe^{2+}]\times0.07} \quad \text{or} \quad [Fe^{2+}] = \frac{10^{-17}}{1.6\times0.07} = \frac{10^{-15}}{11.2} = 8.928\times10^{-17}$$

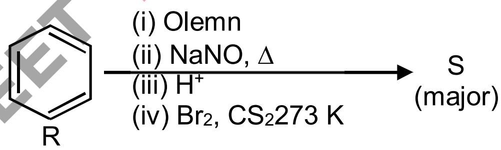
or $8.93 \times 10^{-17} = Y \times 10^{-17}$

32. Scheme 1 and 2 describe the conversion of P to Q and R to S, respectively. Scheme 3 describes the synthesis of T from Q and S. The total number of Br atoms in a molecule of T is ________

Scheme 1 :



Scheme 2 :

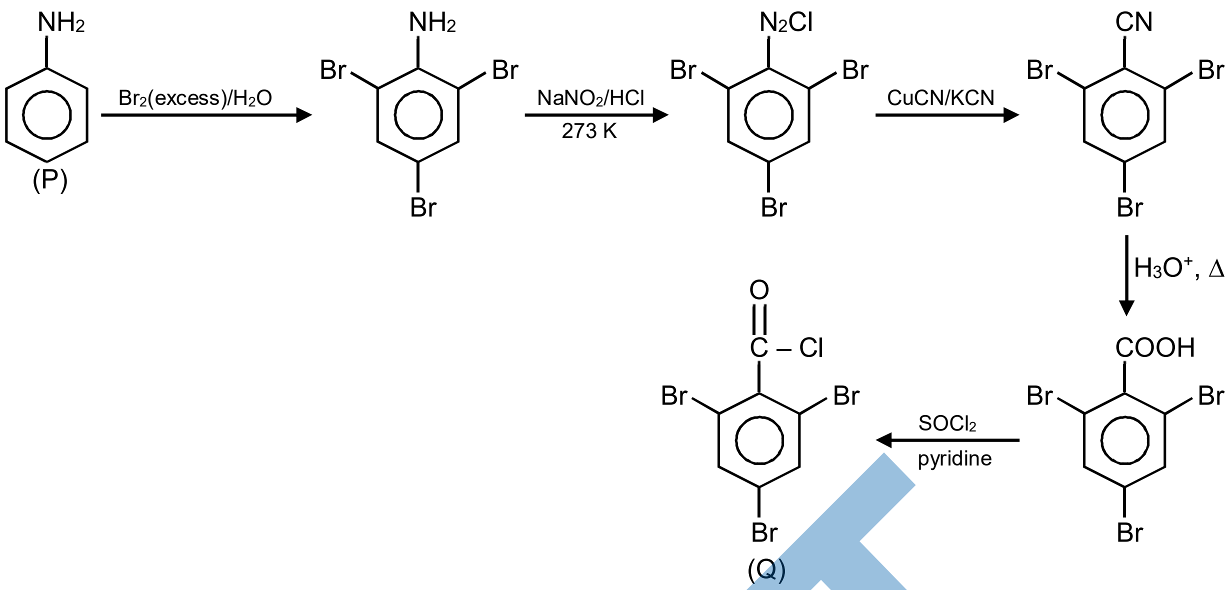


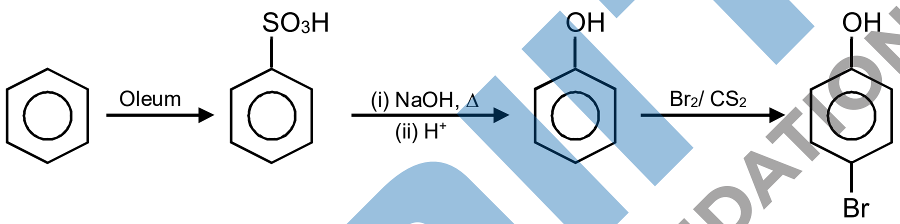
Scheme 3 :

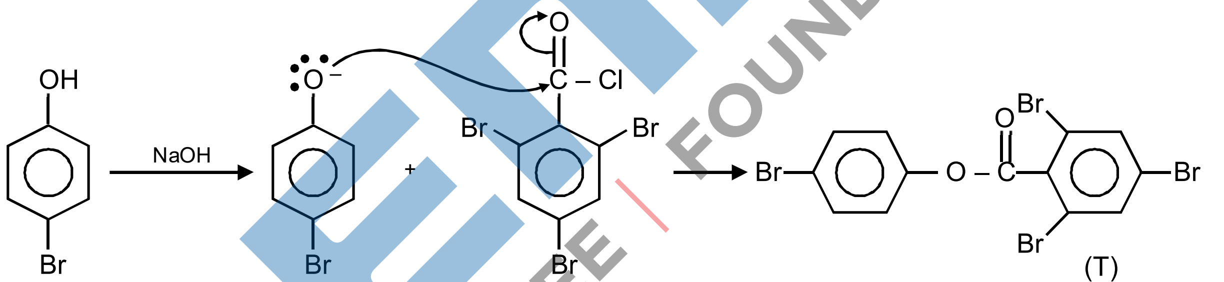
$$S \xrightarrow[\text{(ii) Q}]{\text{(i) NaOH}} \underset{\text{(major)}}{T}$$

Ans. 4.00

Sol.







33. On dissolving 0.5 g of a non-volatile non-ionic solute to 39 g of benzene, its vapor pressure decreases from 650 mm Hg to 640 mm Hg. The depression of freezing point of benzene (in K) upon addition of the solute is ______

(Given data : Molar mass and the molal freezing point depression constant of benzene are 78 g mol^{-1} and 5.12 K kg mol^{-1}, respectively)

Ans. 1.02

Sol. $\frac{p_0 - p_s}{p_s} = i\left(\frac{n_{solute}}{n_{solvent}}\right)$

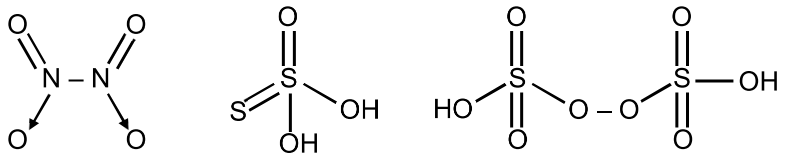
$\frac{650-640}{640} = 1 \times \frac{0.5 \times 78}{M \times 39}$

$\Delta T_f - K_f \times \text{molality} = 5.12 \times \frac{0.5 \times 1000}{64 \times 39}$

$\Delta T_f = 1.02$

34. Among B_2H_6, $B_3N_3H_6$, N_2O, N_2O_4, $H_2S_2O_3$ and $H_2S_2O_8$, the total number of molecules containing covalent bond between two atoms of the same kind is__________

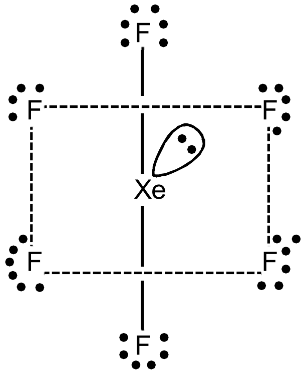
Ans. 4.00

Sol. 

35. At 143 K. the reaction of XeF_4 with O_2F_2 produces a xenon compound Y. The total number of lone pair(s) of electrons present on the whole molecule of Y is ______

Ans. 19.00

Sol. $XeF_4 + O_2F_2 \longrightarrow XeF_6 + O_2$



36. Consider the kinetic data given in the following table for the reaction A + B + C ⟶ Product

Experiment	[A] (mol dm^{-3})	[B] (mol dm^{-3})	[C] (mol dm^{-3})	Rate of reaction (mol dm^{-3} s^{-1})
1	0.2	0.1	0.1	6.0×10^{-5}
2	0.2	0.2	0.1	6.0×10^{-5}
3	0.2	0.1	0.2	1.2×10^{-4}
4	0.3	0.1	0.1	9.0×10^{-5}

The rate of the reaction for [A] = 0.15 mol dm^{-3}, [B] = 0.25 mol dm^{-3} and [C] = 0.15 mol dm^{-3} is found to be Y × 10^{-5} mol dm^{-3} s^{-1}. The value of Y is ___________

Ans. 6.75

Sol. Rate k $[A]^x [B]^y [C]^z$

By exp. No. 1 & 2 $\quad y = 0$

By exp. No. 1 & 3 $\quad z = 1$

By exp. No. 1 & 4 $\quad x = 1$

Rate = k $[A]^1 [B]^0 [C]^1$

From Exp. No.1 $6 \times 10^{-5} = k\,(0.2)\,(0.1)$

$\Rightarrow k = 3 \times 10^{-3}$

Now for [A] = 0.15 $\quad$ [B] = 0.25 $\quad$ [C] = 0.15

Rate = k $[A]^1 [B]^0 [C]^1$

$= 3 \times 10^{-3} \times 0.15 \times 1 \times 0.15$

$= 3 \times 0.0225 \times 10^{-3} = \underbrace{6.75}_{Y} \times 10^{-5}\,\text{molL}^{-1}\,\text{sec}^{-1}$

PART-C : MATHEMATICS

SECTION-1 : (Maximum Marks : 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options. ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : +3 If ONLY the correct option is chosen.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered)

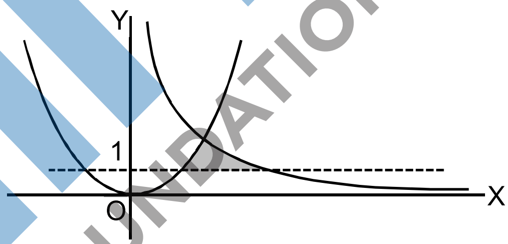
Negative Marks : –1 In all other cases

37. The area of the region $\{(x, y) : xy \le 8,\ 1 \le y\ x^2\}$ is

(A) $16\ell og_e 2–6$ (B*) $16\ell og_e 2-\frac{14}{3}$ (C) $8\ell og_e 2-\frac{14}{3}$ (D) $8\ell og_e 2-\frac{7}{3}$

Ans. B

Sol. For intersection, $\frac{8}{y}=\sqrt{y} \Rightarrow y = 4$



Hence, required area = $\int_1^4\left(\frac{8}{y}-\sqrt{y}\right)dy$

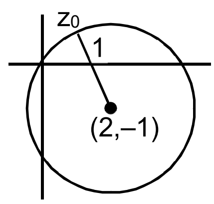
$=\left[8\ell n y-\frac{2y^{\frac{3}{2}}}{3}\right]_1^4 = 16\ell n2-\frac{14}{3}$.

38. Let S be the set of all complex numbers z satisfying $|z-2+i| \ge \sqrt{5}$. If the complex number z_0 is such that $\frac{1}{|z_0-1|}$ is the maximum of the set $\left\{\frac{1}{|z-1|}: z \in S\right\}$, then the principal argument of $\frac{4-z_0-\bar{z}_0}{z_0-\bar{z}_0+2i}$ is

(A) $\frac{\pi}{2}$ (B) $\frac{3\pi}{4}$ (C) $\frac{\pi}{4}$ (D*) $\frac{-\pi}{2}$

Ans. D

Sol. $\arg\left(\frac{4-(z_0-\bar{z}_0)}{(z_0-\bar{z}_0)+zi}\right) = \arg\left(\frac{4-2\mathrm{Re}\,z_0}{2i\mathrm{Im}\,z_0+2i}\right)$o



$\arg\left(\frac{2-\mathrm{Re}\,z_0}{(1+\mathrm{Im}\,z_0)i}\right) = \arg\left(-\left(\frac{2-\mathrm{Re}\,z_0}{1+\mathrm{Im}\,z_0}\right)i\right)$

= arg (– ki), k > 0 [As Re z_0 < 2 and Im z_0 > 0]

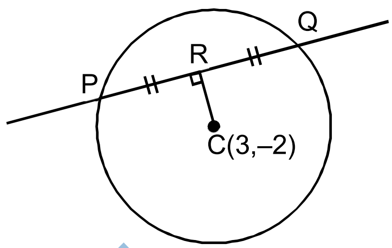
$= \frac{-\pi}{2}$

39. A line y = mx + 1 intersects the circle $(x – 3)^2 + (y + 2)^2 = 25$ at the points P and If the midpoint of the line segment PQ has x-coordinate $\frac{-3}{5}$, then which one of the following options is correct?

(A) $-3 \le m < -1$ (B) $4 \le m < 6$ (C*) $2 \le m < 4$ (D) $6 \le m < 8$

Ans. C

Sol. $R = \left(\frac{-3}{5}, \frac{-3m}{5} + 1\right)$



So, $m\left(\frac{\frac{-3m}{5}+3}{\frac{-3}{5}-3}\right) = -1$

$\Rightarrow m^2 - 5m + 6 = 0$

$\Rightarrow m = 2, 3.$

40. Let $M = \begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$,

where $\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ are real numbers, and I is the 2 × 2 identity matrix. If

α^* is the minimum of the set $\{\alpha(\theta) : \theta \in [0,2\pi]$, and

β^* is the minimum of the set $\{\beta(\theta) : \theta \in [0,2\pi)\}$,

then the value of $\alpha^* + \beta^*$ is

(A) $\frac{-37}{16}$ (B) $\frac{-31}{16}$ (C) $\frac{-17}{16}$ (D*) $\frac{-29}{16}$

Ans. D

Sol. Given $M = \alpha I + \beta M^{-1}$

$\Rightarrow M^2 - \alpha M - \beta I = O$

By putting values of M and M^2, we get

$\alpha(\theta) = 1 - 2\sin^2\theta\cos^2\theta = 1 - \frac{\sin^2 2\theta}{2} \ge \frac{1}{2}$

Also, $\beta(\theta) = -(\sin^4\theta\cos^4\theta + (1+\cos^2\theta)(1+\sin^2\theta))$

$= -(\sin^4\theta\cos^4\theta + 1 + \cos^2\theta + \sin^2\theta + \sin^2\theta\cos^2\theta)$

$= -(t^2 + t + 2),\ t = \frac{\sin^2 2\theta}{2} \in \left[0, \frac{1}{4}\right]$

$\Rightarrow \beta(\theta) \ge \frac{-37}{16}$

SECTION-2 : (Maximum Marks: 32)

– This section contains EIGHT (08) questions.

– Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).

– For each question, choose the option(s) corresponding to (all) the correct answer(s).

– Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is (are) chosen.

Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.

Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct.

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : –1 In all other cases.

– For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;

choosing ONLY (A) and (B) will get +2 marks;

choosing ONLY (A) and (D) will get +2 marks;

choosing ONLY (B) and (D) will get +2 marks;

choosing ONLY (A) will get +1 marks;

choosing ONLY (B) will get +1 marks;

choosing ONLY (D) will get +1 marks;

choosing no option (i.e. the question is unanswered) will get 0 marks, and

choosing any other combination of options will get –1 mark.

41. In a non-right-angled triangle ΔPQR, let p, q, r denote the lengths of the sides opposite to the angles at P, Q, R respectively. The median from R meets the side PQ at S, the perpendicular from P meets the side QR at E, and RS and PE intersect at O. If p = $\sqrt{3}$, q = 1, and the radius of the circumcircle of the ΔPQR equals 1, then which of the following options is/are correct ?

(A) Area of Δ SOE = $\frac{\sqrt{3}}{12}$

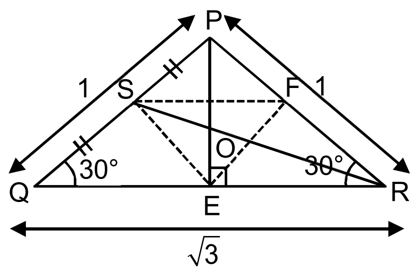
(B*) Radius of incircle of ΔPQR = $\frac{\sqrt{3}}{2}\left(2-\sqrt{3}\right)$

(C*) Length of RS = $\frac{\sqrt{7}}{2}$

(D*) Length of OE = $\frac{1}{6}$

Ans. (B, C, D)

Sol. $\frac{\sin P}{\sqrt{3}}=\frac{\sin Q}{1}=\frac{1}{2R}=\frac{1}{2}$



$\Rightarrow P=\frac{\pi}{3}$ or $\frac{2\pi}{3}$ and $Q=\frac{\pi}{6}$ or $\frac{5\pi}{6}$

Since, p > q ⇒ P > Q

So, if P=$\frac{\pi}{3}$and $Q=\frac{\pi}{6}\Rightarrow R=\frac{\pi}{2}$(not possible)

Hence, P = $\frac{2\pi}{3}$ and $Q=R=\frac{\pi}{6}$

$$r=\frac{\Delta}{s}=\frac{\frac{1}{2}(1)\left(\sqrt{3}\right)\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}+2}{2}\right)}=\frac{\sqrt{3}}{2}(2-\sqrt{3})$$

Now, area of $\Delta SEF = \frac{1}{4}$ area of ΔPQR

$\Rightarrow$ area of $\Delta SOE = \frac{1}{3}$ area of $\Delta SEF = \frac{1}{12}$ area of $\Delta PQR = \frac{1}{12}\cdot\frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{48}$

$$RS = \frac{1}{2}\sqrt{2(3)+2(1)-1} = \frac{\sqrt{7}}{2}$$

$$OE = \frac{PE}{3} = \frac{1}{3}\cdot\frac{1}{2}\sqrt{2(1)^2+2(1)^2-3} = \frac{1}{6},.$$

42. Let $f : R \to R$ is given by

$$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0 \\ x^2 - x + 1, & 0 \le x < 1 \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3}, & 1 \le x < 3 \\ (x-2)\ell og_e(x-2) - x + \frac{10}{3}, & x \ge 3 \end{cases}$$

Then which of the following options is/are correct?

(A*) f ' has a local maximum at x = 1

(B*) f is onto

(C) f is increasing on $(-\infty, 0)$

(D*) f ' is NOT differentiable at x = 1

Ans. A, B, D

Sol.
$$f(x) = \begin{cases} (x+1)^5 - 2x, & x < 0 \\ x^2 - x + 1, & 0 \le x < 1 \\ \frac{2x^3}{3} - 4x^2 + 7x - \frac{8}{3}, & 1 \le x < 3 \\ (x-2)\ell og_e(x-2) - x + \frac{10}{3}, & x \ge 3 \end{cases}$$

For x < 0, f(x) is continuous

and $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to 0^-} f(x) = 1$

Hence, $(-\infty, 1) \subset$ Range of f(x) in $(-\infty, 0)$

$f'(x) = 5(x+1)^4 - 2$, which changes sign in $(-\infty, 0)$

$\Rightarrow$ f(x) is non-monotonic in $(-\infty, 0)$

For $x \ge 3$, f(x) is again continuous and $\lim_{x\to\infty} f(x) = \infty$ and $f(3) = \frac{1}{3}$

$\Rightarrow \left[\frac{1}{3}, \infty\right) \subset$ Range of f(x) in $[3, \infty)$

Hence, range of f(x) in R

$$f'(x) = \begin{cases} 2x - 1, & 0 \le x < 1 \\ 2x^2 - 8x + 7, & 1 \le x < 3 \end{cases}$$

Hence f ' has a local maximum at x = 1 and f ' is NOT differentiable at x = 1.

43. There are three bags B_1, B_2 and B_3. The bag B_1 contains 5 red and 5 green balls, B_2 contains 3 red and 5 green balls, and B_3 contains 5 red and 3 green balls. Bags B_1, B_2 and B_3 have probabilities $\frac{3}{10}, \frac{3}{10}$ and $\frac{4}{10}$ respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct?

(A) Probability that the selected bag is B_3, given that the chosen balls is green, equals $\frac{5}{13}$.

(B*) Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$.

(C*) Probability that the chosen ball is green equals $\frac{39}{80}$.

(D) Probability that the selected bag is B_3 and the chosen ball is green equals $\frac{3}{10}$.

Ans. (B, C)

Sol.

Ball	Balls composition	$P(B_i)$
B_1	$5R + 5G$	$\frac{3}{10}$
B_2	$3R + 5G$	$\frac{3}{10}$
B_3	$5R + 3G$	$\frac{4}{10}$

(A) $P(B_3 \cap G) = P(B_3) = P\left(\frac{G_1}{B_3}\right)P(B_3) = \frac{3}{8} \times \frac{4}{10} = \frac{3}{20}$

(B) $P(G) = P\left(\frac{G_1}{B_1}\right)P(B_1) + p\left(\frac{G}{B_2}\right)P(B_2) = p\left(\frac{G}{B_3}\right)P(B_3) = \frac{3}{20} + \frac{3}{16} + \frac{3}{20} = \frac{39}{80}$

(C) $P\left(\frac{G}{B_3}\right) = \frac{3}{8}$

(D) $P\left(\frac{B_3}{G}\right) = \frac{P(G \cap B_3)}{P(G)} = \frac{3/20}{39/80} = \frac{4}{13}$.

44. Let Γ denote a curve $y = y(x)$ which is in the first quadrant and let the point (1, 0) lie on it. Let the tangent to Γ at a point P intersect the y-axis at Y_P. If PY_P has length 1 for each point P on Γ, then which of the following options is/are correct?

(A) $y = -\ell og_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) + \sqrt{1-x^2}$

(B*) $y = \ell og_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}$

(C) $xy' - \sqrt{1-x^2} = 0$

(D*) $xy' + \sqrt{1-x^2} = 0$ $xy' + \sqrt{1-x^2} = 0$

Ans. (B, D)

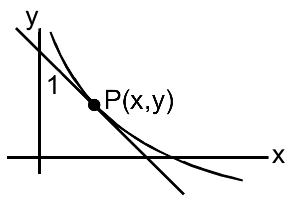
Sol. $Y - y = y'(X - x)$

So, $Y_P = (0, y - xy')$

So, $x^2 + (xy')^2 = 1 \Rightarrow \frac{dy}{dx} = -\sqrt{\frac{1-x^2}{x^2}}$

[$\frac{dy}{dx}$ cannot be positive i.e. f(x) cannot be increasing in first quadrant for $x \in (0, 1)$]

Hence, $\int dy = -\int \frac{\sqrt{1-x^2}}{x} dx$



$\Rightarrow y = -\int \frac{\cos^2\theta}{\sin\theta} d\theta$; put $x = \sin\theta$

$\Rightarrow y = -\int \text{cosec}\,\theta\, d\theta + \int \sin\theta\, d\theta$

$\Rightarrow y = \ln(\text{cosec}\,\theta + \cot\theta) - \cos\theta + C$

$\Rightarrow y = \ell n\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2} + C$

$\Rightarrow y = \ell n\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}$ [As y(A) = 0]

45. Let α and β be the roots of $x^2 - x - 1 = 0$, with $\alpha > \beta$. For all positive integers n, define

$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, n \geq 1$

$b_1 = 1$ and $b_n = a_{n-1} + a_{n+1}$, .$n \geq 2$

Then which of the following options is/are correct?

(A*) $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$

(B*) $b_n = \alpha^n + \beta^n$ for all $n \geq 1$

(C) $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

(D*) $a_1 + a_2 + a_3 + \ldots.. + a_n = a_{n+2} - 1$ for all n1

Ans. (A, B, D)

Sol. α, β are roots of $x^2 - x - 1$

$a_{r+2} - a_r = \frac{(\alpha^{r+2} - \beta^{r+2}) - (\alpha^r - \beta^r)}{\alpha - \beta} = \frac{(\alpha^{r+2} - \alpha^r) - (\beta^{r+2} - \beta^r)}{\alpha - \beta}$

$= \frac{\alpha^r(\alpha^2 - 1) - \beta^r(\beta^2 - 1)}{\alpha - \beta} = \frac{\alpha^r\alpha - \beta^r\beta}{\alpha - \beta} = \frac{\alpha^{r+1} - \beta^{r+1}}{\alpha - \beta} = a_{r+1}$

$\Rightarrow a_{r+2} - a_{r+1} = a_r$

$\Rightarrow \sum_{r=1}^{n} a_r = a_{n+2} - a_2 = a_{n+2} - \frac{\alpha^2 - \beta^2}{\alpha - \beta} = a_{n+2} - (\alpha + \beta) = a_{n+2} - 1$

Now, $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{\sum_{n=1}^{\infty}\left(\frac{\alpha}{10}\right)^n - \sum_{n=1}^{\infty}\left(\frac{\beta}{10}\right)^n}{\alpha - \beta} = \frac{\frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} - \frac{\frac{\beta}{10}}{1-\frac{\beta}{10}}}{\alpha - \beta} = \frac{\frac{\alpha}{10-\alpha} - \frac{\beta}{10-\beta}}{\alpha - \beta}$

$$= \frac{10}{(10-\alpha)(10-\beta)} = \frac{10}{89}$$

$$\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \sum_{n=1}^{\infty} \frac{a_{n-1}+a_{n+1}}{10^n} = \frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} + \frac{\frac{\beta}{10}}{1-\frac{\beta}{10}} = \frac{12}{89}$$

Further, $b_n = a_{n+1} + a_{n+1} = \frac{(\alpha^{n-1}-\beta^{n-1})+(\alpha^{n+1}-\beta^{n+1})}{\alpha-\beta}$

(as $\alpha\beta = -1 \Rightarrow \alpha^{n-1} = -\alpha^n\beta$ and $\beta^{n-1} = -\alpha\beta^n$)

$$= \frac{\alpha^n(\alpha-\beta)+(\alpha-\beta)\beta^n}{\alpha-\beta} = \alpha^n + \beta^n.$$

46. Let L_1 and L_2 denote the lines

$\vec{r} = \hat{i} + \lambda(-\hat{i}+2\hat{j}+2\hat{k}),\ \lambda \in R$ and $\vec{r} = \mu(2\hat{i}-\hat{j}+2\hat{k}),\ \mu \in R$

respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3?

(A) $\vec{r} = t(2\hat{i}+2\hat{j}-\hat{k}),\ t \in R$

(B*) $\vec{r} = \frac{2}{9}(2\hat{i}-\hat{j}+2\hat{k}) + t(2\hat{i}+2\hat{j}-\hat{k}),\ t \in R$

(C*) $\vec{r} = \frac{1}{3}(2\hat{i}+\hat{k}) + t(2\hat{i}+2\hat{j}-\hat{k}),\ t \in R$

(D*) $\vec{r} = \frac{2}{9}(4\hat{i}+\hat{j}+\hat{k}) + t(2\hat{i}+2\hat{j}-\hat{k}),\ t \in R$

Ans. (B, C, D)

Sol. Points on L_1 and L_2 are respectively A $(1-\lambda, 2\lambda, 2\lambda)$ and B $(2\mu, -\mu, 2\mu)$

So, $\overrightarrow{AB} = (2\mu+\lambda-1)\hat{i} + (-\mu-2\lambda)\hat{j} + (2\mu-2\lambda)\hat{k}$

and vector along their shortest distance $= 2\hat{i}+2\hat{j}-\hat{k}$

Hence, $\frac{2\mu+\lambda-1}{2} = \frac{-\mu-2\lambda}{2} = \frac{2\mu-2\lambda}{-1}$

$= \lambda = \frac{1}{9}$ and $\mu = \frac{2}{9}$

Hence, $A \equiv \left(\frac{8}{9}, \frac{2}{9}, \frac{2}{9}\right)$ and $B \equiv \left(\frac{4}{9}, \frac{-2}{9}, \frac{4}{9}\right)$

$\Rightarrow$ Mid-point of AB $\equiv \left(\frac{2}{3}, 0, \frac{1}{3}\right)$.

47. Let M = an $\begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$ and adj M $= \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$ where a and b are real numbers. Which of the following options is/are correct?

(A*) a + b = 3

(B*) $(\text{adj } M)^{-1} + \text{adj } M^{-1} = -M$

(C*) If $M\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then $\alpha-\beta+\gamma = 3$

(D) $\det(\text{adj } M^2) = 81$

Ans. (A, B, C)

Sol. $(\text{adj } M)_{11} = 2 - 3b = -1 \Rightarrow b = 1$

Also, $(\text{adj } M)_{22} = -3a = -6 \Rightarrow a = 2$

Now, $\det M = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix} = -2$

$\Rightarrow \det(\text{adj } M^2) = (\det M^2)^2 = (\det M)^4 = 16$

Also, $M^{-1} = \dfrac{\text{adj} M}{\det M}$

$\Rightarrow \text{adj } M = -2M^{-1} \Rightarrow (\text{adj } M)^{-1} = \dfrac{-M}{2}$

and, $\text{adj } (M)^{-1} = (M^{-1})^{-1} \det(M^{-1}) = \dfrac{M}{\det M} = \dfrac{-M}{2}$

Hence, $(\text{adj } M)^{-1} + \text{adj } (M^{-1}) = -M$

Further, MX = b

$$\Rightarrow X = M^{-1}b = \frac{-\text{adj}M}{2} b = \frac{-1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \frac{-1}{2}\begin{bmatrix} -2 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$\Rightarrow (\alpha, \beta, \gamma) = (1, -1, 1)$

48. Define the collections $\{E_1, E_2, E_3, \ldots\ldots\}$ of ellipses and $\{R_1, R_2, R_3, \ldots\ldots\}$ of rectangles as follows:

$E_1 : \dfrac{x^2}{9} + \dfrac{y^2}{4} = 1;$

R_1 : rectangle of largest area, with sides parallel to the axes, inscribed in E_1 ;

E_n : ellipse $\dfrac{x^2}{a_n^2} + \dfrac{y^2}{b_n^2} = 1$ of largest area inscribed in R_{n-1}, n > 1 ;

R_n : rectangle of largest area, with sides parallel to the axes, inscribed in E_n, n > 1.

Then which of the following options is/are correct?

(A*) The length of latus rectum of E_9 is $\dfrac{1}{6}$.

(B*) $\sum_{n=1}^{N}(\text{area of } R_n) < 24$, for each positive integer N

(C) The eccentricities of E_{18} and E_{19} are NOT equal.

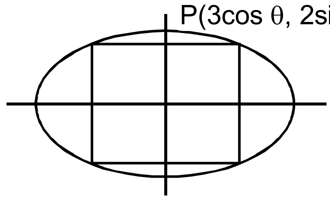
(D) The distance of a focus from the centre in E_9 is $\dfrac{\sqrt{5}}{32}$.

Ans. (1, 2)

Sol. Area of R_1 = 3sin 2θ, for this to be maximum

$\Rightarrow \theta = \dfrac{\pi}{4} \Rightarrow \left(\dfrac{3}{\sqrt{2}}, \dfrac{2}{\sqrt{2}}\right)$

Hence for subsequent areas of rectangles R_n to be maximum the coordinates will be in G.P. with common ratio $r = \frac{1}{\sqrt{2}} \Rightarrow a_n = \frac{3}{\left(\sqrt{2}\right)^{n-1}}$; $b_n = \frac{3}{\left(\sqrt{2}\right)^{n-1}}$



Eccentricity of all the ellipses will be same

Distance of a focus from the centre in $E_9 = a_9e_9 = \sqrt{a_9^2 - b_9^2} = \frac{\sqrt{5}}{16}$

Length of latus rectum of $E_9 = \frac{2b_9^2}{a_9} = \frac{1}{6}$

$$\sum_{n=1}^{\infty} \text{Area of } R_n = 12 + \frac{12}{2} + \frac{12}{4} + \ldots\ldots \infty = 24$$

$$\Rightarrow \sum_{n=1}^{N} (\text{area of } R_n) < 24, \text{ for each positive integer N}$$

SECTION-3 : (Maximum Marks: 18)

– This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE.

– For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.

– Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct numerical value is entered.

Zero Marks : 0 In all other cases.

49. Let S be the sample space of all 3 × 3 matrices with entries from the set {0, 1}. Let the events E_1 and E_2 be given by

$E_1 = \{A \in S : \det A = 0\}$ and

$E_2 = \{A \in S : \text{sum of entries of A is } 7\}$.

If a matrix is chosen at random from S, then the conditional probability $P(E_1|E_2)$ equals ____

Ans. 0.50

Sol. $n(E_2) = {}^9C_2$ (as exactly two cyphers are there)

Now, det A = 0, when two cyphers are in the same column or same row

$\Rightarrow n(E_1 \cap E_2) = 6 \times {}^3C_2$

Hence, $P\left(\frac{E_1}{E_2}\right) = \frac{n(E_1 \cap E_2)}{n(E_2)} = \frac{18}{36} = \frac{1}{2} = 0.50$

50. Three lines are given by

$\vec{r} = \lambda\hat{i}, \lambda \in R$

$\vec{r} = \mu(\hat{i} + \hat{j}), \mu \in R$ and

$\vec{r} = \nu(\hat{i} + \hat{j} + \hat{k}), \nu \in R$

Let the lines cut the plane x + y + z = 1 at the points A, B and C respectively. If the area of the triangle ABC is Δ then the value of $(6\Delta)^2$ equals ___

Ans. 0.75

Sol. A (1, 0, 0), B $\left(\frac{1}{2},\frac{1}{2},0\right)$ and C$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

Hence, $\overrightarrow{AB}=\frac{-\hat{i}}{2}+\frac{\hat{j}}{2}$ and $\overrightarrow{AC}=\frac{-2\hat{i}}{3}+\frac{\hat{j}}{3}+\frac{\hat{k}}{3}$

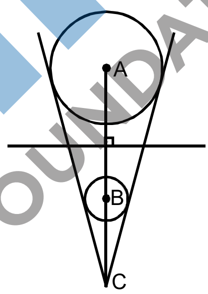
So, $\Delta=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|=\frac{1}{2}\sqrt{\frac{1}{2}\times\frac{2}{3}-\frac{1}{4}}=\frac{1}{2\times2\sqrt{3}}$

$\Rightarrow (6\Delta)^2=\frac{3}{4}=0.75.$

51. Let the point B be the reflection of the point A(2, 3) with respect to the line 8x – 6y – 23 = 0. Let Γ_A and Γ_A be circles of radii 2 and 1 with centres A and B respectively. Let T be a common tangent to the circles Γ_A and Γ_B such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is _____

Ans. 10.00

Sol. Distance of point A from given line = $\frac{5}{2}$



$\frac{CA}{CB}=\frac{2}{1}$

$\Rightarrow \frac{AC}{AB}=\frac{2}{1}$

$\Rightarrow$ AC = 2 × 5 = 10.

52. Let AP (a; d) denote the set of all the terms of an infinite arithmetic progression with first term a and common difference d > 0. If AP (1; 3) $\cap$ AP (2; 5) $\cap$ AP (3; 7) = AP(a; d) then a + d equals ___.

Ans. 157.00

Sol. We equate the general terms of three respective

A.P.'s as 1 + 3a = 2 + 5b = 3 + 7c

$\Rightarrow$ 3 divides 1 + 2b and 5 divides 1 + 2c

$\Rightarrow$ 1 + 2c = 5, 15, 25 etc.

So, first such terms are possible when 1 + 2c = 15 i.e. c = 7

Hence, first term a = 52

d = lcm (3, 5, 7) = 105

$\Rightarrow$ a + d = 157.

53. If $I=\frac{2}{\pi}\int_{-\pi/4}^{\pi/4}\frac{dx}{(1+e^{\sin x})(2-\cos 2x)}$ then $27I^2$ equals _____

Ans. 4.00

Sol. $2I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4}\left(\frac{1}{(1+e^{\sin x})(2-\cos 2x)}+\frac{1}{(1+e^{-\sin x})(2-\cos 2x)}\right)dx$

Using King's rule

$\Rightarrow I = \frac{1}{\pi}\int_{-\pi/4}^{\pi/4}\frac{dx}{2-\cos 2x} = \frac{2}{\pi}\int_{0}^{\pi/4}\frac{dx}{2-\cos 2x} = \frac{2}{\pi}\int_{0}^{\pi/4}\frac{\sec^2 x}{1+3\tan^2 x}dx$

$= \frac{2}{\sqrt{3}\pi}\left[\tan^{-1}\left(\sqrt{3}\tan x\right)\right]_0^{\pi/4} = \frac{2}{3\sqrt{3}}$

$\Rightarrow 27I^2 = 27 \times \frac{4}{27} = 4.$

54. Let $\omega \neq 1$ be a cube root of unity. Then the minimum of the set

$\{|a+b\omega+c\omega^2|^2 : a, b, c \text{ distinct non-zero integers}\}$

equals ______

Ans. 3.00

Sol. $| a + b\omega + c\omega^2 |^2 = (a + b\omega + c\omega^2)$

$= (a + b\omega + c\omega)^2 (a + b\omega^2 + c\omega) = a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2] \geq \frac{1+1+4}{2} = 3$ (when a = 1, b = 2, c = 3).